Serb. Astron. J. № 167 (2003), 75 - 80

UDC 52–17
Preliminary report

MULTI-COMPOSING OF THE ORDINATES AS A SPECTRAL FILTER

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(Received: October 6, 2003; Accepted: October 17, 2003)

SUMMARY: In this paper we describe a method of plain and multi composing of ordinates to define spectral filters. We apply the method to the simulated equidistant observations, and we find that the suitable filters are easy to construct and apply.

Key words. Methods: data analysis - Methods: numerical

1. INTRODUCTION

There are several common limitations of the spectral analysis methods: incommensurability of a considered periodic component and of the interval of observations; non-equidistant registration of dependant variable; unknown starting points of the periodic terms; existence and level of modulation (damping of components); limited interval of the observations.

Practice suggests that there is no universal method to overcome these limitations. If we do not have enough theoretical points for the analytical modelling of observational data, for a particular observational material, we must choose a corresponding method.

Earth's rotation is disturbed and influenced by many forces producing different changes. We have fixed our attention on the periodic changes and their long-period fluctuations.

In an attempt to select suitable spectral filters to examine real observational data, we have here synthesized data that correspond to observations. In particular, we generated the data similar to observational data from the Time Service of the Belgrade Astronomical Observatory.

Then, in accordance with our experience in using the method of multi-composing of ordinates, as described in Đurović (1979), we constructed some suitable spectral filters for the decomposition of the synthetic data.

2. PLAIN AND COMPLEX TRANSFORMATIONS

Assume that observational data are given by a table $y_i = f(x_i), (i = 1, n)$, where $x_{i+1} - x_i = const.$; if the process contains only harmonic terms, then

$$y_i \approx \sum_{p=1}^{m} A_p \cos(\omega_p x_i + \Phi_p)$$

where A_p is the amplitude of harmonic component, ω_p is the so called circular frequency ($\omega = 2\pi/T_p$) and Φ_p is its phase. Then

$$y_i = \sum_{p=1}^{m} A_p \cos(\omega_p x_i + \Phi_p) + s(x_i)\,, \qquad (1)$$

where $s(x_i)$ is the noise.

The first task in the spectral analysis is obtaining of the unknown parameters $A_p, \omega_p, \Phi_p, p = 1, m$ for all m harmonic components. To achieve this one finds *selective transformation* (**spectral filter**) for a given p, with minimum number of steps needed for optimization. It means that a chosen filter must preserve only useful transformed values.

Let $n = 2r + 1$, where r is an integer, and $y_i, i = 1, n$ is sorted by increasing x. For all $l \ll r$ one selects a set $y_{k\pm j}, j = 0, l, l \leq k \leq 2r + 1 - l$; let

$$R_l = c_0 y_k + \sum_{j=1}^{l} c_j (y_{k-j} + y_{k+j})\,, \tag{2}$$

or

$$R_l = c_0 Y_k + \sum_{j=1}^{l} c_j Y_j\,, \tag{3}$$

where $Y_j^k = y_{k-j} + y_{k+j}$ for $j = 1, l$. R_l is plain (linear) transformation of order l, with coefficients c_j in the neighborhood of the period $T_j = 2\pi/\omega_j$ (Đurović 1979).

Under assumption that in Eq. (1) $s(x) = 0$, we have

$$\begin{aligned} Y_j^k &= \sum_{p=1}^{m} \{A_p \cos[\omega_p (x_k - j) + \Phi_p] \\ &+ A_p \cos[\omega_p (x_k + j) + \Phi_p]\}\,, \end{aligned}$$

or

$$Y_j^k = \sum_{p=1}^{m} 2\cos(\omega_p j) A_p \cos(\omega_p x_k + \Phi_p). \tag{4}$$

The series, resulting from (4), is harmonic process with same phases and frequencies, but with different amplitudes; this difference is measured by

$$\alpha_j^p = 2\cos(\omega_p j) \tag{5}$$

By substituting this in Eq. (3) we obtain a new expression for the linear transformation

$$R_l = \sum_{p=1}^{m} (c_0 + \sum_{j=1}^{l} c_j \alpha_j^p A_p \cos(\omega_p x_k + \Phi_p). \tag{6}$$

This means that the amplitudes of harmonic components of basic signal are changed by a quantity

$$\rho_k^p = c_0 + \sum_{j=1}^{l} c_j \alpha_j^p\,, \tag{7}$$

known as power factor. Set of these $R \equiv \{\rho_k^p, k = 1, l\}$ is known as *filter*, originating from $\rho = \rho(\omega)$.

When coefficients $c_j, j = 1, l$ in R_l are equal to $-1,\ 0,\ 1$, then these are plain or simple combinations of ordinates with low selectivity.

To eliminate the low selectivity one has to sum plain transformations.

Let

$$S_l = y_k + \sum_{j=1}^{l} Y_j$$

be such transformation. Power-factors are given by

$$\sigma_l^p = 1 + \sum_{j=1}^{l} \alpha_j^p = \frac{\cos \frac{2l+1}{p}\pi}{\cos \frac{\pi}{p}}.$$

The choice of l is done in accordance with the condition $\sigma_l^p = 0$ for the corresponding ω_p, i.e. p. The differentiation will yield conditions for the equilibrium points

$$\frac{\partial \sigma_l^p}{\partial p} = 0\,, \tag{8}$$

providing that $\cos(\pi/p) \neq 0$.

Of special significance are double transformations with arbitrary shift. Let shift be $L/2$. Then

$$(S_l)_{L/2} = y_k + \sum_{j=1}^{l} (-1)^j Y_{jL/2}\,, \tag{9}$$

and power factor is

$$(\sigma_l^p)_{L/2} = \pm \frac{\cos \frac{2l+1}{p} \frac{L\pi}{2}}{\cos \frac{1}{p} \frac{L\pi}{2}}.$$

Roots of $(\sigma_l^p)_{L/2}$ are given by

$$p = \frac{(2l+1)L}{2P+1}; \quad p \neq \frac{L}{2P+1}, \quad P = 0, 1, 2, ...$$

This transformation will amplify the harmonic term with period $p = L$ and its odd subharmonic terms.

Finally, we can construct multi-transformation for the case of distinguishing very close periodic components.

As we have seen, one maximum of power factors in the case of double transformation is at infinity and is thus independent of l. By composing these double transformations it is possible to damp all periodic terms, except these at infinity:

$$\Pi(Y) = Y_l Y_s ... Y_t\,,$$

and new power factor is

$$\tau(\alpha) = \alpha_l \alpha_s ... \alpha_t. \tag{10}$$

If $l < s < ... < t$, boundary of amplification is $p_1 = 4t$. The others result from

$$\frac{t}{2} \leq l \leq t, \; za \;\; \Pi(Y) = Y_l Y_t\,,$$

$$\frac{3t}{7} \le l \le s-1, \ \ \frac{3t}{5} \le s \le t-1, \ \ za\, \Pi(Y) = Y_l Y_s Y_t\,,$$

$$\frac{2t}{5} \le l \le s-1, \qquad \frac{t}{2} \le s \le q-1, \ \ \frac{2t}{3} \le q \le t-1,$$
$$za\, \Pi(Y) = Y_l Y_s Y_q Y_t\,,$$

and so on.

In order to damp terms $p_i \le p_1$, zeros of power factor $\tau(\alpha)$ must be uniformly distributed in the region $p_i \le p_1$,

$$p = \frac{4l}{2L+1}, \ \ p = \frac{4s}{2L+1}, ..., p = \frac{4t}{2L+1}.$$

Similar procedure applies to the other kinds of the multi-transformations.

$$\Pi(S) = S_l S_s ... S_t$$

with power factors

$$\tau(\sigma) = \sigma_l \sigma_s ... \sigma_t.$$

There are zeros

$$p = \frac{2l+1}{L}, \ \ p = \frac{2s+1}{L}, ... \ \ p = \frac{2t+1}{L}\,,$$

where $p \ne 1/L$, and t is found from

$$t_1 = \frac{p_1 - 1}{2}.$$

Arbitrary shifting in multi-transformations provides the possibility to separate periodic terms very close to a given periodic term. Curves representing power factors of these transformations consist of series with very prominent peaks corresponding to the period $p = P$, and to its odd subharmonics.

The general procedure consists of the following steps. First, we apply the transformation $\Pi(S)$ or similar, to amplify components with $p_i \ge 2t+1$ or $p_i \ge 2t$. Next, we gradually decrease t in order to successively identify the other components. Finally, the identified terms are precisely determined with $\Pi(S_l)_{L/2}$ and $\Pi(Z_l)_{L/2}$, and removed before repetition of the procedure.

3. SYNTHESIS OF POLY-HARMONIC PROCESS AND DECOMPOSITION

We have simulated data $UT1 - UTC$ in an interval of 20 years; all data are made equidistant. The synthesis covers harmonic terms with periods of 1, 1.2, 2.0 and 9.0 years:

$$\begin{aligned} UTS(t) \ &= \ 0.12\sin(\frac{2\pi}{365.25}t + \frac{\pi}{2}) + \\ &+ \ 0.165\sin(\frac{2\pi}{439.30}t + \frac{\pi}{4}) + \\ &+ \ 0.03\sin(\frac{2\pi}{724.5}t + \frac{2\pi}{3}) + \\ &+ \ 0.08\sin(\frac{2\pi}{3287.25}t + \frac{\pi}{6}) \end{aligned}$$

Data are presented on Fig. 1.

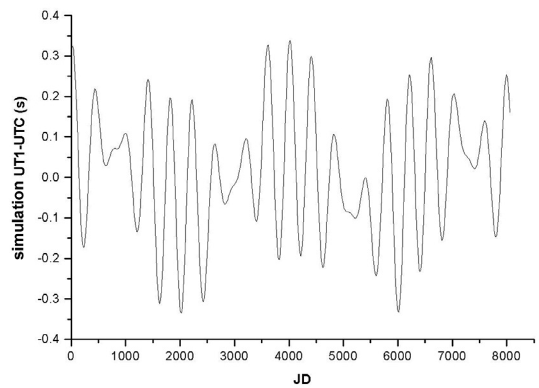

Fig. 1. *Simulated data $UT1 - UTC$ containing harmonic terms with periods of 1.0, 1.2, 2.0 and 9.0 years.*

First, we apply transformation $S_{15}S_{16}S_{17}$. The curve of selectivity is shown on Fig. 2.

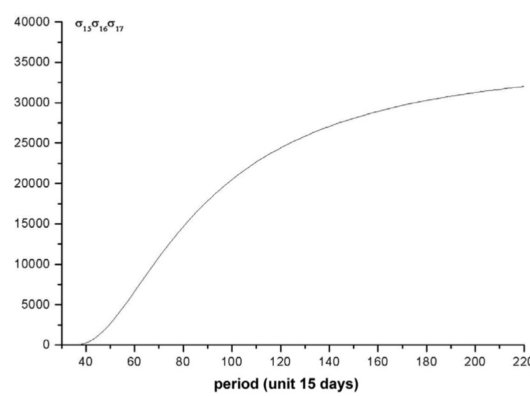

Fig. 2. *Selectivity curve of transformation* $S_{15}S_{16}S_{17}$.

This transformation removes all periodic terms with periods less then $35 \times 15 = 525$ days (≈ 1.5 year), and the 2 year term is damped. The residual signals (henceforth simply referred to as *residuals*) are divided by power factor

$$\sigma_{15}\sigma_{16}\sigma_{17} = \frac{\sin(\frac{31\pi}{9\times 365.25/15}) \times \sin(\frac{33\pi}{9\times 365.25/15})}{\sin^3(\frac{\pi}{9\times 365.25/15})} \times$$
$$\times \sin(\frac{35\pi}{9\times 365.25/15})$$

to derive the true amplitudes. Fig. 3 shows the obtained values with only the 9 year term remaining.

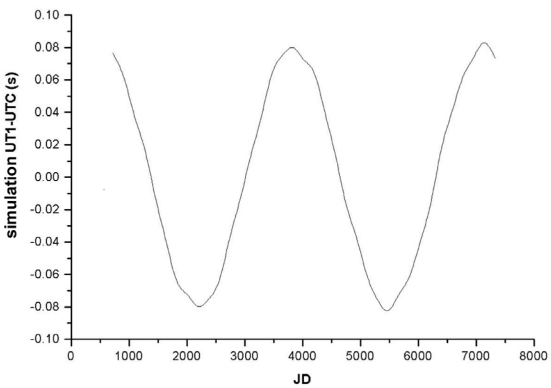

Fig. 3. *Isolated 9 year term.*

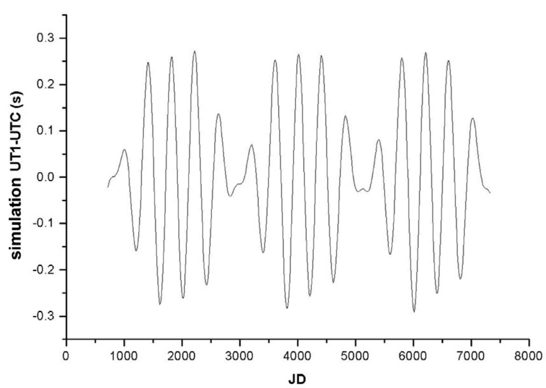

Fig. 4. *Curve derived after removing 9 year term from removal of the $UT1 - UTC$ data.*

After removal of even the 9 year term (residuals plotted on Fig. 4), we apply transformation $s_{13}s_{14}s_{15}$ which damped all periodic terms with periods less then 465 days (≈ 1.3 years). The curve of selectivity is represented on Fig. 5.

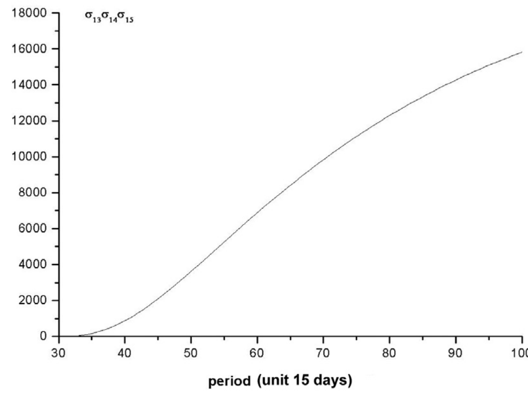

Fig. 5. *Selectivity curve of transformation $S_{13}S_{14}S_{15}$.*

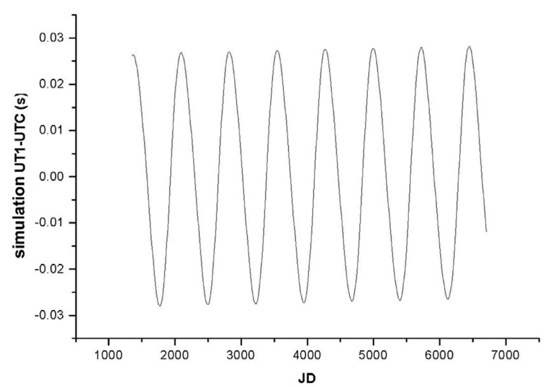

Fig. 6. *Isolated 2 year term.*

Fig. 6 shows isolated 2 year term with amplitude derived by dividing residuals with power factor

$$\sigma_{13}\sigma_{14}\sigma_{15} = \frac{\sin(\frac{27\pi}{2\times 365.25/15}) \times \sin(\frac{29\pi}{2\times 365.25/15})}{\sin^3(\frac{\pi}{2\times 365.25/15})} \times$$
$$\times \sin(\frac{31\pi}{2\times 365.25/15})$$

After removal of the 2 year term (see Fig. 7), we have applied to the residuals the transformation $(s_4s_6)_{24/2}$ with maximum at annual term and zero at Chandler nutation position (Fig. 8).

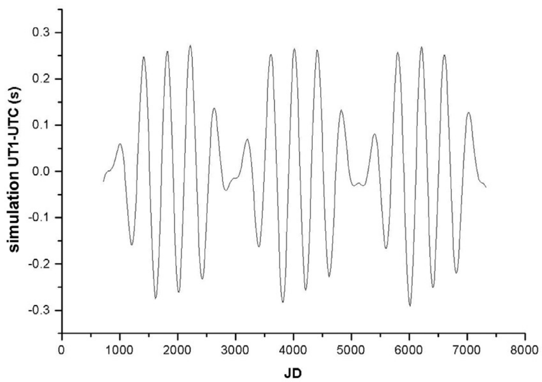

Fig. 7. *Curve derived after removing 9 year term and 2 year term from removal of the $UT1-UTC$ data.*

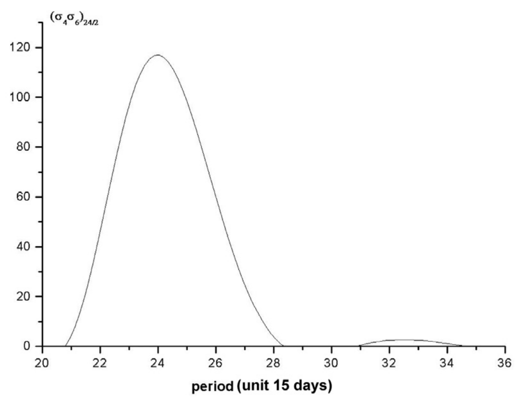

Fig. 8. *Selectivity curve of transformation* $(S_4S_6)_{24/2}$.

Power factor of annual term (see Fig. 9) is given by

$$(\sigma_4\sigma_6)_{24/2} = \frac{\cos(\frac{9\times24\pi}{2\times365.25/15})\times\cos(\frac{13\times24\pi}{2\times365.25/15})}{\cos^2(\frac{24\pi}{2\times365.25/15})}$$

After removal of the annual term, the residuals correspond to the 1.2 year term only (see Fig. 10).

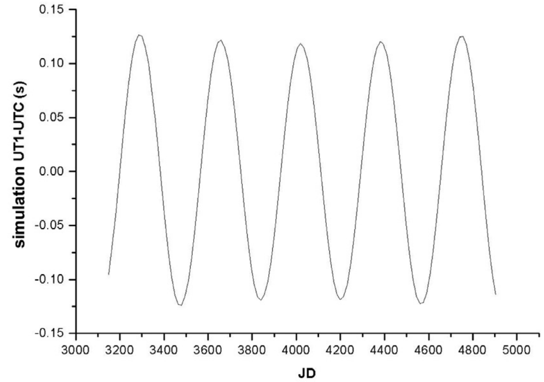

Fig. 9. *Isolated annual term.*

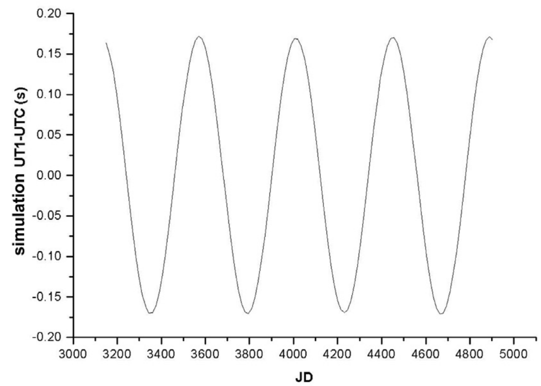

Fig. 10. *Isolated 1.2 year term.*

4. CONCLUSIONS

Under conditions assumed to hold in the synthesis of model data, spectral filters derived from multi-composing of ordinates are easy to construct and apply.

The real observational data are, however, different from the simulated ones, and we must test our filters especially in the case of latitude and time service observations.

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ВИШЕСТРУКЕ ТРАНСФОРМАЦИЈЕ ОРДИНАТА КАО СПЕКТРАЛНИ ФИЛТЕР

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UDK 52–17
Претходно саопштење

У овом раду описујемо метод простих и вишеструких ордината за дефинисање спектралних филтера. Применили смо метод на симулирана еквидистантна посматрања и нашли смо погодне филтере који се лако конструишу и употребљавају.